Dynamical phase transitions study in simulations of finite neurons network

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Introduction → The Model

Ferrari et al. [1] introduced a continuous-time interacting-point-processes model for network of spiking neurons with two states: "active" or "quiescent" (1 or 0).
The neuron goes from "active" to "quiescent" either when it spikes, or by the effect of the leakage. The opposite happens whenever a neighbor neurons spikes.
- The spikes are modeled as the events of a Poisson process of parameter 1;
- The leakage events are modeled as the events of a Poisson process of some positive parameter gamma γ.

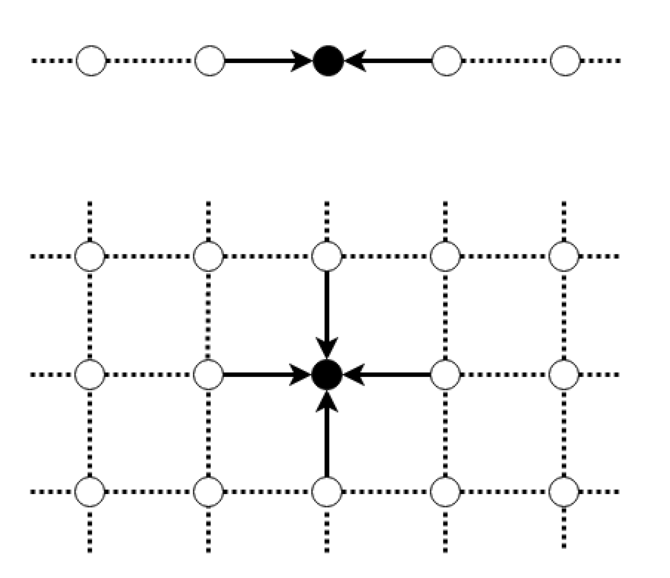

Figure 1:One-dimensional and two-dimensional lattices. A directed arrow is drawn toward the black neuron from each of its presynaptic neurons.

In [1] it was shown that this model presents phase transition with respect to the parameter γ, that is, exists a parameter γ_C such that:
- For all $\gamma > \gamma_C$, the system will reach a configuration with all neurons in the quiescent state in finite time with probability 1.
- For all $\gamma < \gamma_C$, there is a positive probability that the system will never reach a configuration with all neurons in the 'quiescent' state.

Finite number of neurons

- It is usual to work with a necessarily finite number of neurons when modeling the brain;

AND

- When the number of neurons is finite we know by elementary results about Markov chains that the absorbent state, where all neurons are "quiescent", will necessarily be reached in some finite time for any value of γ.

Conjecture

There exists γ_c such that if $\gamma < \gamma_c$, then we have the following convergence:

$$\lim_{N\to\infty} \Gamma\left(\frac{\sigma_N}{E(\sigma_N)}\right) \to e^{-t}$$

In words, the re-normalized time of extinction σ_N converges in distribution to an exponential random variable of mean 1.

To back up our conjecture we build up the present model in python and run it 10,000 turns for N=(10, 50, 100, 500, 1000), γ=(4.00, 0.85, 0.50, 0.40, 0.35, 0.30) and plot the normalized histogram.

Results

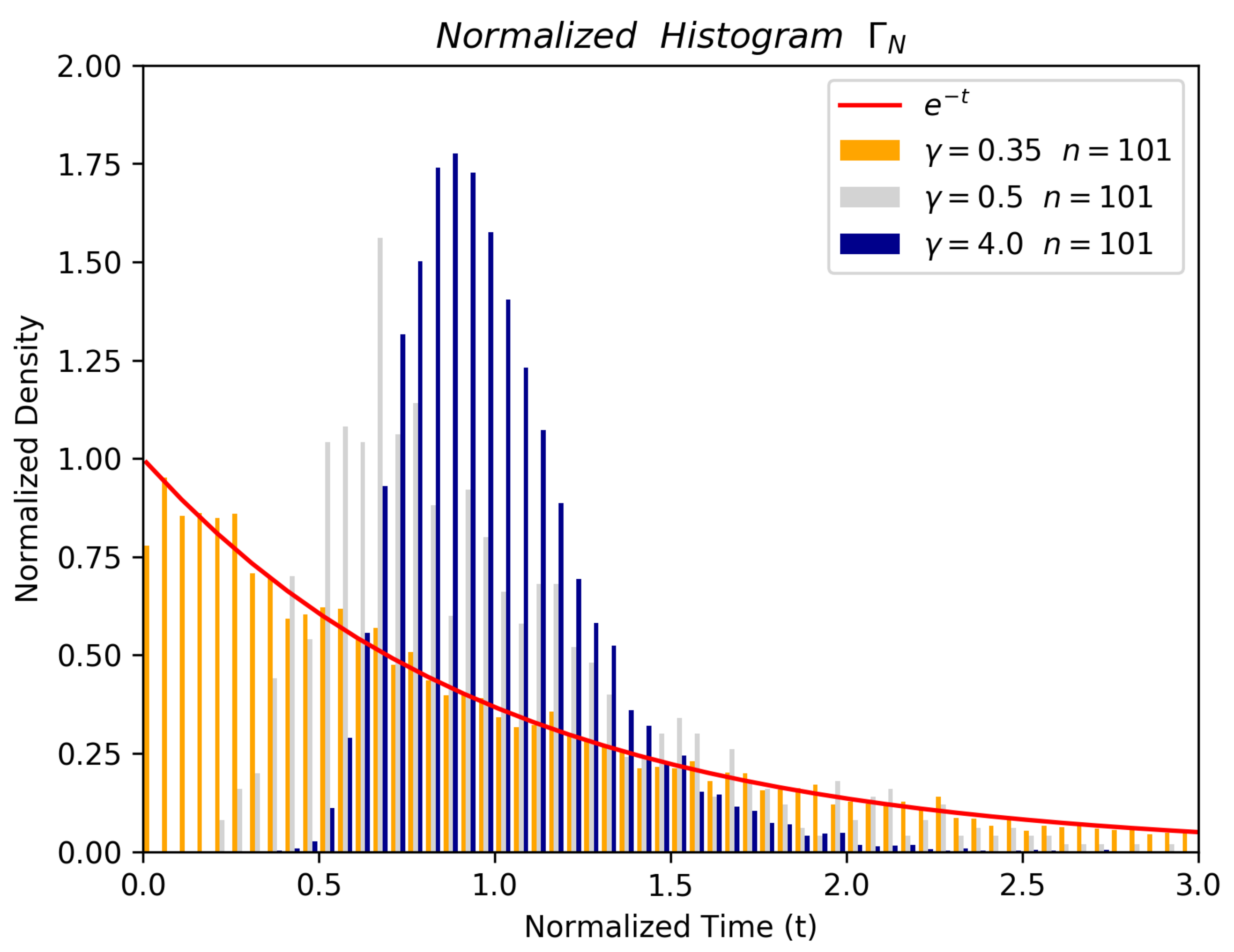

Figure 2: Normalized σ_N behavior for different values of γ for each lattice: The blue, gray and orange bars are histograms of the renormalized time of extinction σ_N in the high γ (super-critical), intermediate γ (due to the finite number of neurons) and low γ (sub-critical) simulations respectively for the case Z^1 and the red line is the exponential e^{-t}.

Table 1:Average of time of extinguishing in the sub-critical case for a different 2N+1 number of neurons and gamma. The mean time of extinguishes increases to lower gamma and higher N.

N	2N+1	Gamma γ 0.40	0.35	0.30
10	21	70.07	144.48	410.70
50	101	442.61	11,104.55	15,009,122.87
100	201	790.03	506,962.32	-
500	1001	1,882.99	-	-
1000	2001	2,261.59	-	-

Other Dimension lattices Z^d

Super-critical case:

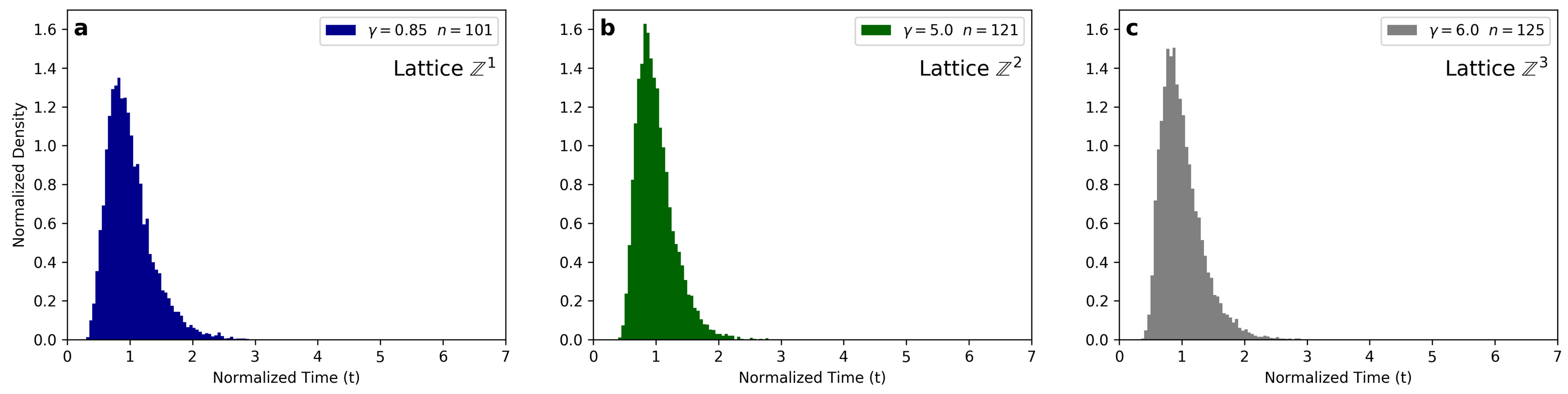

Figure 3: Normalized σ_N sub-critical behavior for different values of γ for each lattice: a, b and c the blue, green and gray bars are the histogram for high γ values (super-critical) simulations also for, respectively, the lattices Z^1, Z^2 and Z^3.

Sub-critical case:

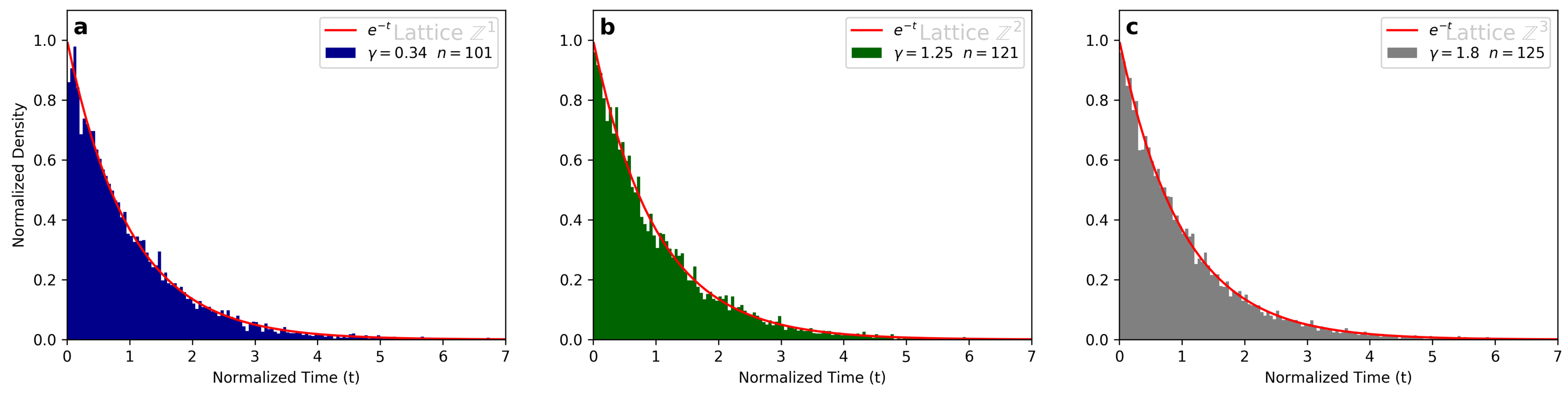

Figure 4: Normalized σ_N sub-critical behavior for different values of γ for each lattice: In a, b and c the blue, green and gray bars are histograms of the renormalized time of extinction σ_N in the low γ (sub-critical) simulations for the cases Z^1, Z^2 and Z^3 respectively and the red line is the exponential e^{-t}.

References

[1] Ferrari PA, Galves A, Grigorescu I et al. Phase transition for infinite systems of spiking neurons. J Stat Phys (2018) 172:1564–1575 https://doi.org/10.1007/s10955-018-2118-6

[2] Cassandro M, Galves A, Picco P. Dynamical phase transitions in disordered systems: the study of a random walk model. InAnnales de l'IHP Physique théorique 1991 (Vol. 55, No. 2, pp. 689-705).

Acknowledgement

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