Introduction → The Model

Ferrari et al. [1] introduced a continuous-time interacting-point-processes model for network of spiking neurons with two states: “active” or “quiescent” (1 or 0). The neuron goes from “active” to “quiescent” either when it spikes, or by the effect of the leakage. The opposite happens whenever a neighbor neurons spikes.

- The spikes are modeled as the events of a Poisson process of parameter 1;
- The leakage events are modeled as the events of a Poisson process of some positive parameter \( \gamma \).

In [1] it was shown that this model presents phase transition with respect to the parameter \( \gamma \), that is, exists a parameter \( \gamma_c \) such that:
- For all \( \gamma > \gamma_c \), the system will reach a configuration with all neurons in the quiescent state in finite time with probability 1.
- For all \( \gamma < \gamma_c \), there is a positive probability that the system will never reach a configuration with all neurons in the ‘quiescent’ state.

Finite number of neurons

- It is usual to work with a necessarily finite number of neurons when modeling the brain;
- When the number of neurons is finite we know by elementary results about Markov chains that the absorbent state, where all neurons are “quiescent”, will necessarily be reached in some finite time for any value of \( \gamma \).

Conjecture

There exists \( \gamma_c \) such that if \( \gamma < \gamma_c \), then we have the following convergence:

\[
\lim_{N \to \infty} \frac{\sigma_N}{E[\sigma_N]} \to e^{-1}
\]

In words, the re-normalized time of extinction \( \sigma_N \) converges in distribution to an exponential random variable of mean 1.

To back up our conjecture we build up the present model in python and run it 10,000 turns for \( N = \{10, 50, 100, 500, 1000\} \), \( \gamma = \{4.00, 0.85, 0.50, 0.40, 0.35, 0.30\} \) and plot the normalized histogram.

Results

Table 1: Average of time of extinguishing in the sub-critical case for different \( 2N+1 \) number of neurons and gamma. The mean time of extinguishing increases to lower gamma and higher \( N \).

<table>
<thead>
<tr>
<th>Gamma ( \gamma )</th>
<th>0.40</th>
<th>0.35</th>
<th>0.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 2N+1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>70.07</td>
<td>144.48</td>
<td>410.70</td>
</tr>
<tr>
<td>50</td>
<td>442.61</td>
<td>506.962.32</td>
<td>11,009,122.87</td>
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<tr>
<td>100</td>
<td>790.03</td>
<td>1,882.99</td>
<td>-</td>
</tr>
<tr>
<td>500</td>
<td>1,882.99</td>
<td>5,069.62</td>
<td>-</td>
</tr>
<tr>
<td>1000</td>
<td>2,261.59</td>
<td>11,104.55</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 1: One-dimensional and two-dimensional lattices. A directed arrow is drawn toward the black neuron from each of its presynaptic neurons.

For all \( \gamma_c \) values (super-critical) simulations also for, respectively, the lattices \( Z_1 \), \( Z_2 \) and \( Z_3 \).

Other Dimension lattices \( Z^d \)

Super-critical case:

Sub-critical case:

Figure 3: Normalized \( \sigma_N \) sub-critical behavior for different values of \( \gamma \) for each lattice: a, b and c the blue, green and gray bars are the histogram for high \( \gamma \) values (super-critical) simulations also for, respectively, the lattices \( Z_1 \), \( Z_2 \) and \( Z_3 \).

Figure 4: Normalized \( \sigma_N \) sub-critical behavior for different values of \( \gamma \) for each lattice: In a, b and c the blue, green and gray bars are histograms of the re-normalized time of extinction \( \sigma_N \) in the low \( \gamma \) (sub-critical) simulations for the cases \( Z_1 \), \( Z_2 \) and \( Z_3 \) respectively and the red line is the exponential \( e^{-1} \).

References


Acknowledgement