A Theoretical Approach to Intrinsic Timescales in Spiking Neural Networks

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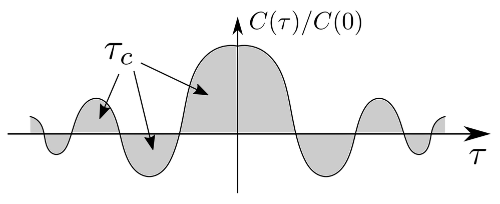
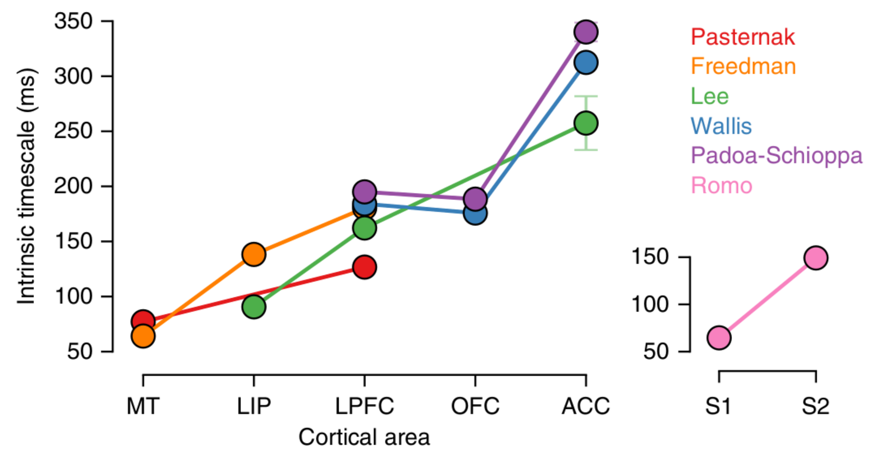
Introduction

- challenge addressed: a **theory of temporal autocorrelations** in spiking neural network models
- particular interest: intrinsic timescales, characterized by **single-unit autocorrelation times** τ_c, in network models with biologically constrained connectivity [1, 2]

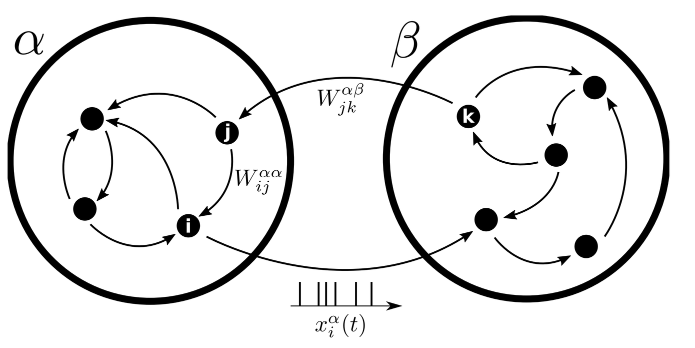


- usually investigated in networks of (non-spiking) rate neurons [3], but in vivo electrophysiological recordings in resting state reveal a hierarchical structure of intrinsic timescales in single unit spiking activity (adapted from [4, Fig. 1d]):



Methods

Dynamic Mean-Field Theory



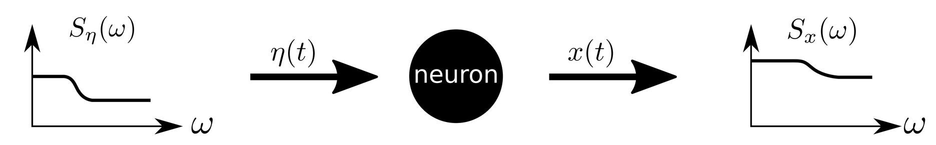
- aim: **coarse grained description**, i.e. one (stochastic) equation per population α instead of one per neuron
- intuition: input $I_{i,\mathrm{in}}^\alpha(t) = \tau_\mathrm{m}^\alpha \sum_\beta \sum_{j=1}^{N^\beta} W_{ij}^{\alpha\beta} x_j^\beta(t-\tau_\mathrm{del}^{\alpha\beta})$ resembles random process due to randomly weighted sum
- $W_{ij}^{\alpha\beta} = J_{ij}^{\alpha\beta} K_{ij}^{\alpha\beta}$ contains both the synaptic weights $J_{ij}^{\alpha\beta}$ and the connectivity matrix $K_{ij}^{\alpha\beta} \in \{0,1\}$
- formally: input approximated by independent Gaussian processes $I_{i,\mathrm{in}}^\alpha(t) \approx \tau_\mathrm{m}^\alpha \eta_i^\alpha(t)$ with stationary statistics

$$\text{mean:}\quad \mu^\alpha = \sum_\beta \langle W^{\alpha\beta}\rangle_J N^\beta \langle x^\beta\rangle_{\eta^\beta}$$

$$\text{corr.:}\quad C_{\eta^\alpha}(\tau) = \sum_\beta \langle (\Delta W^{\alpha\beta})^2\rangle_J N^\beta \langle x^\beta x^\beta\rangle_{\eta^\beta}(\tau)$$

- substantiated by path-integral methods [5, 6]: characteristic functional $\to$ disorder average $\to$ Hubbard-Stratonovich transformation $\to$ saddle point approximation (exact for infinite network sizes)

Colored Noise Problem



- given the statistics of the input $\eta(t)$ (i.e. μ and $S_\eta(\omega)$), what are the statistics of output $x(t)$ (rate ν and $S_x(\omega)$)?
- Wiener-Khinchin theorem: spectrum $S(\omega)$ equals Fourier transformed autocorrelation function $C(\tau)$
- leads to **self-consistency problem**:
  - dynamic mean-field theory: output statistics $\to$ input statistics
  - colored noise problem: input statistics $\to$ output statistics
- open challenge for many neuron models (but see [7, 8])

Results: IF Networks

Network Model

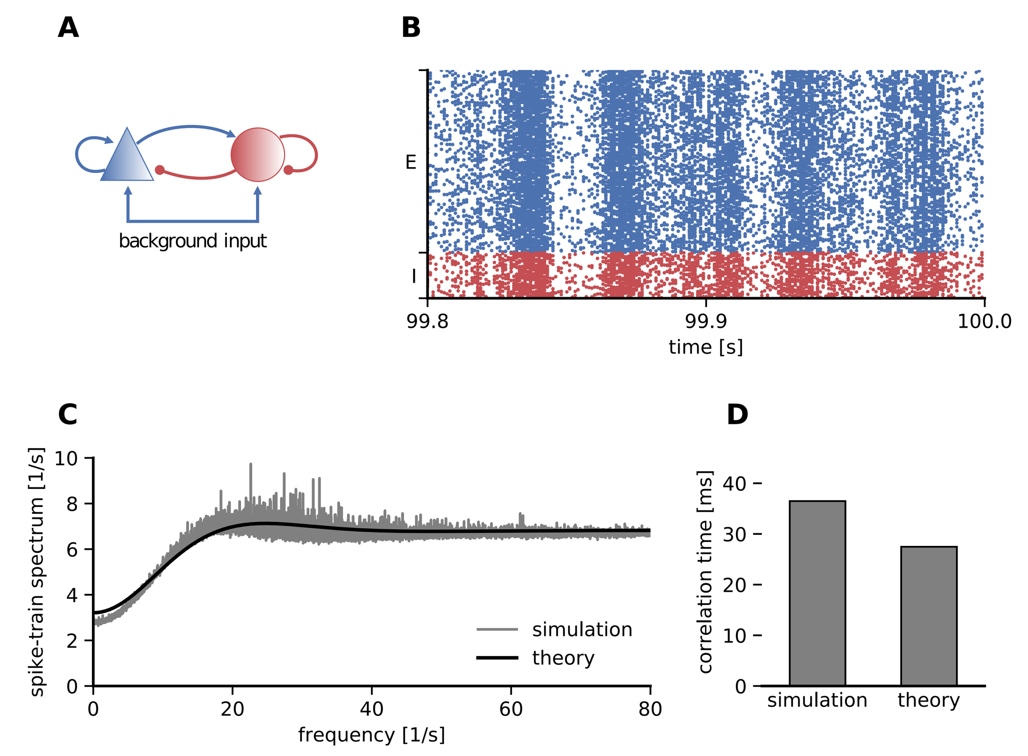
- leaky integrate-and-fire neurons with exponential current-based synapses:

$$\tau_\mathrm{m}^\alpha \dot{V}_i^\alpha = -V_i^\alpha + I_i^\alpha,$$

$$\tau_\mathrm{s}^\alpha \dot{I}_i^\alpha = -I_i^\alpha + \tau_\mathrm{m}^\alpha \sum_\beta \sum_{j=1}^{N^\beta} W_{ij}^{\alpha\beta} x_j^\beta(t-\tau_\mathrm{del}^{\alpha\beta})$$

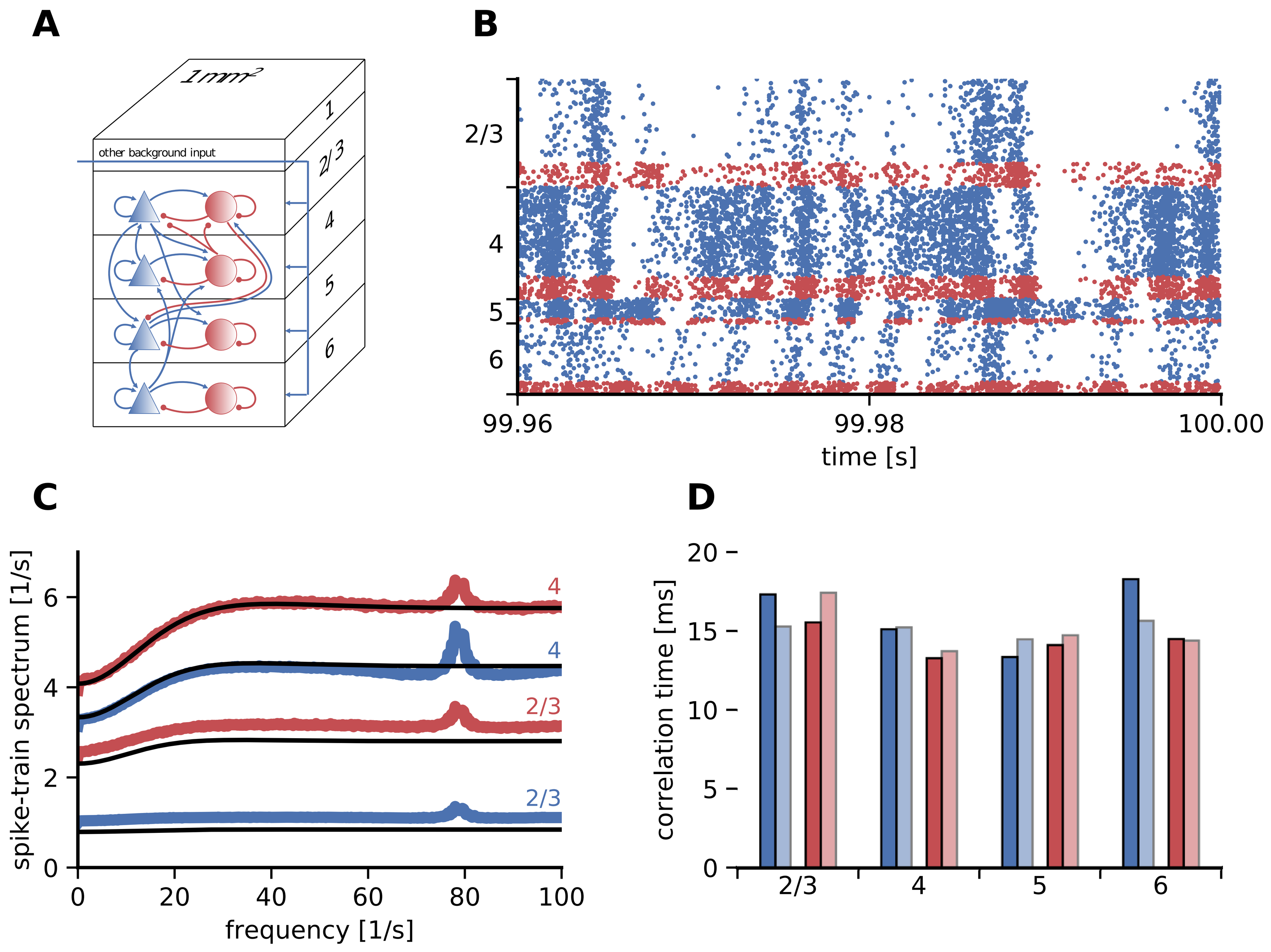
  with fire-and-reset mechanism and refractory period
- Erdős-Rényi topology: $W_{ij}^{\alpha\beta}$ are i.i.d. random variables with cumulants $\langle W^{\alpha\beta}\rangle_J$, $\langle (\Delta W^{\alpha\beta})^2\rangle_J, \ldots$
- external input modeled by Poisson process
- approximate solution of colored noise problem:
  - 1st approximation: output spike train is a renewal process
  - 2nd approximation: hazard function given by the free diffusive flux across the threshold
  - 3rd approximation: firing rate does not change due to the timescales in the input

Balanced Network



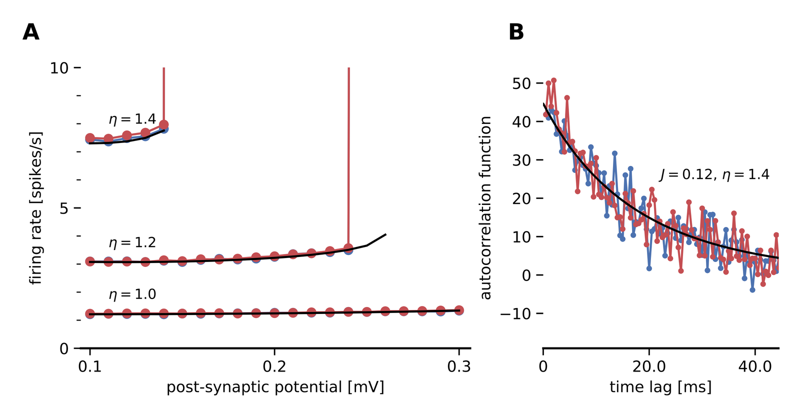
Sketch (**A**) of a balanced spiking neural network [11] with populations of excitatory (blue) and inhibitory (red) neurons. A raster plot (**B**) shows asynchronous irregular dynamics with statistically equivalent neurons. The Fourier transform of the autocorrelation function, i.e. the power spectrum, obtained from our theory (**C**, black line) agrees well with simulations (**C**, gray line). Accordingly, the predicted intrinsic timescale is also in good agreement (**D**).

Structured Network



Sketch of the spiking neural network model with biologically constrained connectivity which integrates knowledge from more than 50 experimental papers (**A**, figure adapted from [1]). A raster plot (**B**) shows asynchronous irregular dynamics with clear statistical differences between the populations. Spike-train power spectra obtained from our theory (**C**, black lines) agree well with simulations except for the peaks around 80 Hz (**C**, colored lines). Here, we selected populations with excellent agreement (layer 4) and with deviations from the theory (layer 2/3). Accordingly, the predicted intrinsic timescales (**D**, shaded bars) are also in good agreement with simulations (**D**, filled bars) where the quantitative agreement depends on the considered population. To account for the peaks in the power spectra and the resulting changes in intrinsic timescales, finite size corrections that take cross-correlations into account are necessary.

Results: GLM networks



In a balanced spiking neural network [11] with populations of excitatory (blue) and inhibitory (red) generalized linear model neurons, rate (**A**) and autocorrelation function (**B**) from the theory (black) agree very well with simulations.

- generalized linear model neurons:

$$V_i^\alpha(t) = \int ds\, \kappa^\alpha(t-s) \sum_\beta \sum_{j=1}^{N^\beta} W_{ij}^{\alpha\beta} x_j^\beta(s)$$

$$\lambda_i^\alpha(t) = c_1^\alpha \exp[c_2^\alpha(V_i^\alpha(t) - V_\mathrm{thresh.}^\alpha)]$$

  where $\lambda_i^\alpha(t)$ is the intensity of Poisson process $x_i^\alpha(t)$
- advantage: colored noise problem analytically solvable
- disadvantage: inherently stochastic neuron dynamics

Discussion

Summary

- for networks of rate units, dynamic mean field theory has yielded significant insights into the interrelation between network structure and intrinsic timescales [3, 7, 13]
- we extend the results to spiking neural networks
  - leaky integrate-and-fire neurons: theory agrees well with simulations in the fluctuation-driven regime
  - generalized linear model neurons: exact analytical solution enables exploration of full parameter space

Outlook

- establishing a link between the connectivity and the emergent intrinsic timescales allows for a thorough investigation of the effect of network architecture
- could be used to fine-tune network models [2] to match the experimentally observed hierarchy of timescales
- focusing on computational aspects, diverse timescales strongly enhance the computational capacity [14]

References

[1] Potjans TC, Diesmann M (2014) *Cereb Cortex* 24 (3): 785-806.
[2] Schmidt M, Bakker R, Hilgetag CC, Diesmann M, van Albada SJ (2018) *Brain Struct Func* 223.3.
[3] Chaudhuri R, Knoblauch K, Gariel MA, Kennedy H, Wang XJ (2015) *Neuron* 88: 419-431.
[4] Murray JD, Bernacchia A, Freedman DJ, Romo R, Wallis JD, Cai X, Padoa-Schioppa C, Pasternak T, Seo H, Lee D, Wang X-J (2014) *Nature Neuroscience* 17.12: 1661-1663.
[5] Sompolinsky H, Zippelius A (1982) *Phys Rev B* 25 (11): 6860-6875.
[6] Schuecker J, Goedeke S, Dahmen D, Helias M (2016) *arXiv* 1605.06758.
[7] Sompolinsky H, Crisanti A, Sommers H (1988) *Phys Rev Lett* 61: 259.
[8] van Meegen A, Lindner B (2018) *Phys Rev Lett* 121: 258302.
[9] Lerchner A, Sterner G, Hertz J, Ahmadi M (2006) *Network: Comp in Neural Systems* 17 (2): 131-150.
[10] Dummer B, Wieland S, Lindner B (2014) *Front Comput Neurosci* 8: 104.
[11] Brunel N (2000) *J Comput Neurosci* 8 (3): 183-208.
[12] Hagen Kleinert. Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets. New Jersey: World Scientific, 2009.
[13] Schuecker J, Goedeke S, Helias M (2018) *Phys Rev X* 8 (4): 041029.
[14] Bellec G, Salaj D, Subramoney A, Legenstein R, Maass W (2018) in *Advances in Neural Information Processing Systems*: 787-797.

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