Maximizing transfer entropy promotes spontaneous formation of assembly sequences in recurrent spiking networks

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Outline

Information theory provides a rich toolbox to understand mechanisms to **process spike sequences** in recurrent neural networks.

Here we apply the **free energy minimization** principle developed by Karl Friston and others to derive learning rules for a spiking sequence learning task.

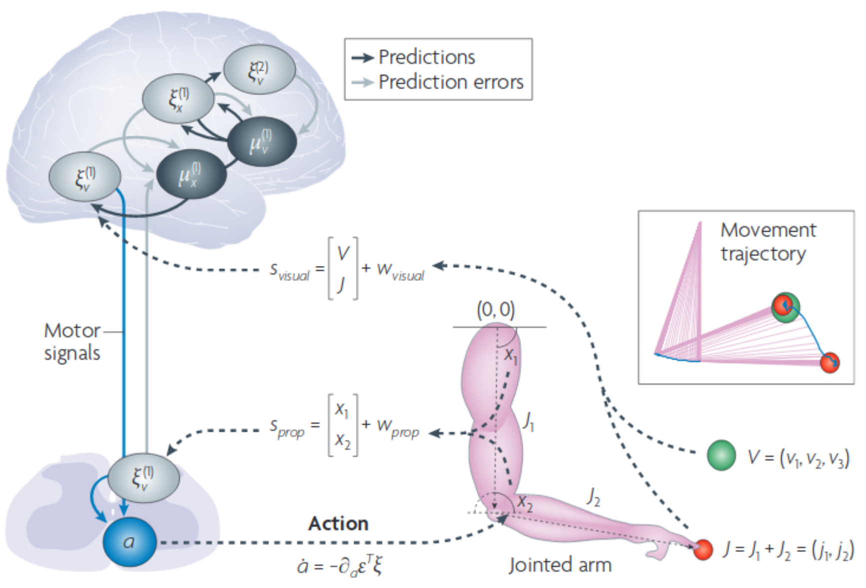
We focus on learning rules that use only **variables that can be accessed locally** at the synapse - the pre-/post-synaptic spikes and the synaptic efficacy.

The derived learning rules resemble experimentally found plasticity mechanisms.

Applying the learning model to a sequence memory task leads to spontaneous formation of **context-specific assembly sequences**.

We analyze the learning model in terms of information transfer within the circuit and find that the learning rules implicitly maximize the **transfer entropy** between neurons.

Introduction: the free energy principle



[Friston, 2010]

Free energy principle in neuroscience:

Minimize the mismatch between your observations and the **internal model of the world** by eliminating the **prediction error**.

This general framework has been applied to motor planning, reasoning, active inference and learning.

Learning in a network means that synaptic updates follow the *gradient of the free energy*:

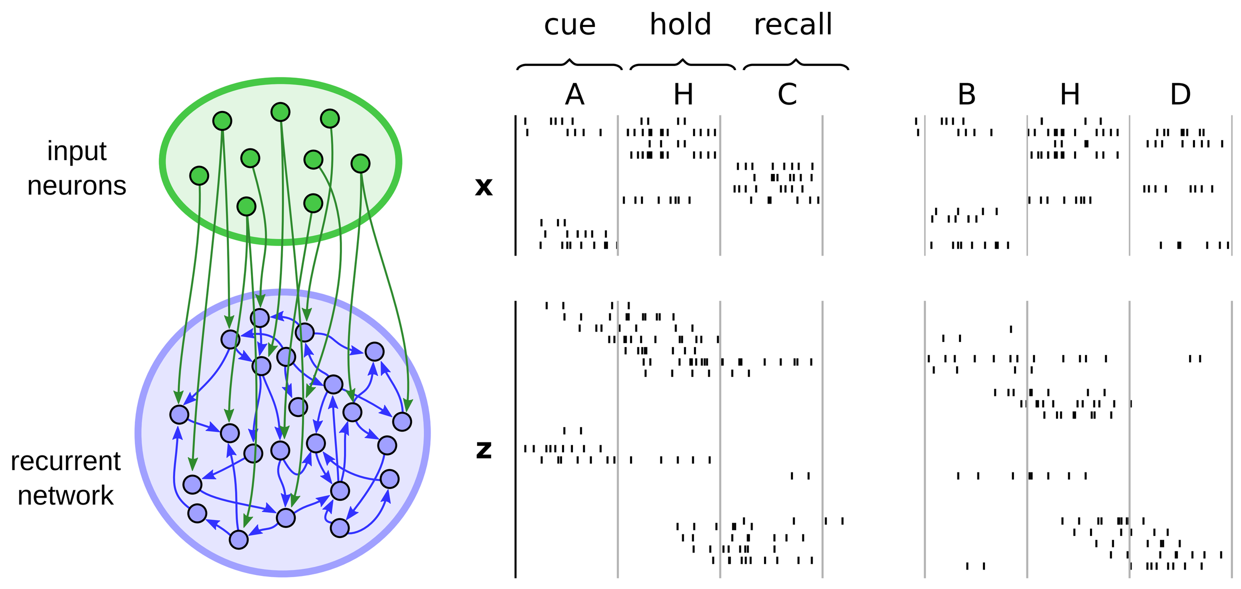
$$\Delta w_{ij} = -\frac{\partial}{\partial w_{ij}}\mathcal{F}$$

Free energy principle for spike sequence learning

Input neurons with high-dimensional spatio-temporal activity **x**.

Responses **z** of recurrent network neurons.

Recurrent network learns to produce actvitiy that encodes **x**.



For the spatiotemporal input sequence **x** and network responses **z**, the free energy is

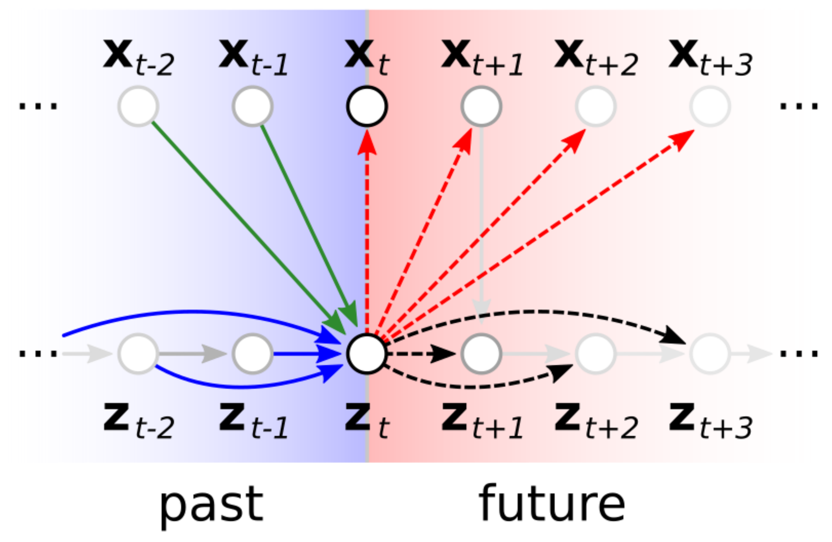
$$\mathcal{F} = -\left\langle q(\mathbf{z}\,|\,\mathbf{x},\boldsymbol{\theta}) \times \log r(\mathbf{x}\,|\,\mathbf{z},\boldsymbol{\theta})\right\rangle_{p^*(\mathbf{x})}$$

Recognition density:

$$q(\mathbf{z}\,|\,\mathbf{x},\boldsymbol{\theta})$$

Prediction density:

$$r(\mathbf{x}\,|\,\mathbf{z},\boldsymbol{\theta})$$



general framework

$$\Delta w_{ij} = -\frac{\partial}{\partial w_{ij}}\mathcal{F}$$

free energy minimization

LIF neuron

membrane potential

$$\rho_k^t = \sum_{i,\tau<t}\epsilon(t-\tau)\,w_{ki}^{(in)}\,x_i^\tau + \sum_{j,\tau<t}\epsilon(t-\tau)\,w_{kj}^{(rec)}\,z_j^\tau + b_k$$

spike generation

$$q(\mathbf{z}\,|\,\mathbf{x},\boldsymbol{\theta}) = \prod_{t,k} H\left(z_k^t\,|\,\rho_k^t\right)$$

spike mechanism $H\left(z_k^t\,|\,\rho_k^t\right)$

lazy prediction

The prediction density reflects neural refractoriness

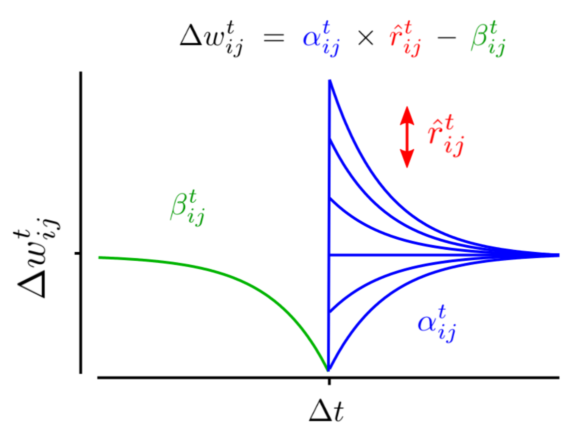
$$r(\mathbf{x}\,|\,\mathbf{z},\boldsymbol{\theta}) \qquad w_{ij}^{(pred)} = -w_{ji}$$

membrane potentials after a post-synaptic spike are promoted to be low.

learning rule

$$\Delta w_{ij}^t = \alpha_{ij}^t \times \hat{r}_{ij}^t - \beta_{ij}^t$$

Free energy minimization enables automatic assembly sequence formation



$$\Delta w_{ij}^t = \underbrace{\alpha_{ij}^t}_{\text{LTP}} \times \underbrace{\hat{r}_{ij}^t}_{\text{mod}} - \underbrace{\beta_{ij}^t}_{\text{LTD}}$$

α_{ij}^t and β_{ij}^t depend only on the pre-/post spike trains.

$\hat{r}_{ij}^t$ depends on pre-/post spiking and the synaptic weight (effectively a regularization term).

We apply this rule to the **sequence memory task** outlined above. Network responses resemble experimentally found assembly sequence *in vivo*, when solving sequence memory tasks **[Harvey, 2012]**.

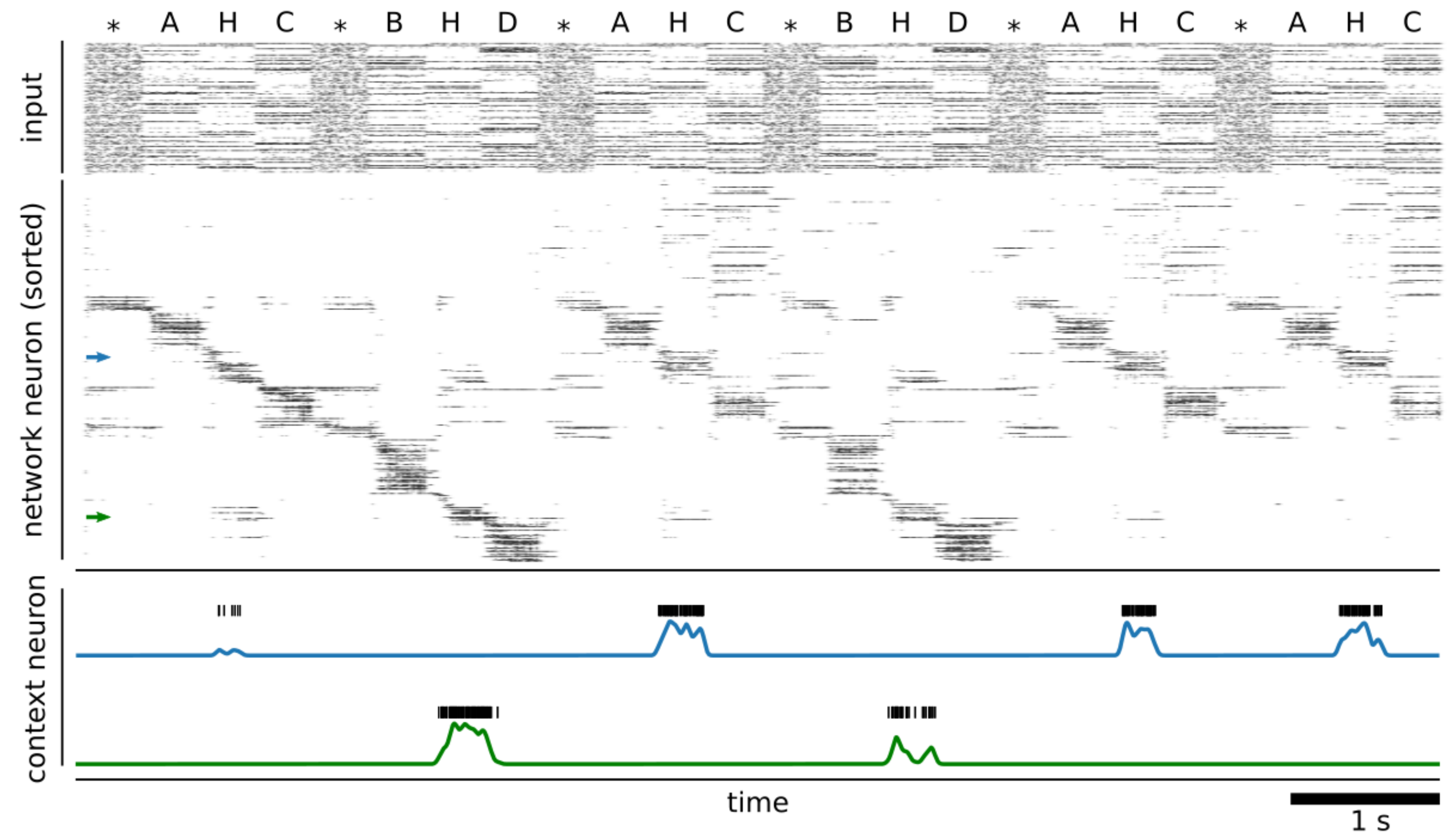


Figure 1: Spontaneous formation of assembly sequences in a sequence memory task

Implicit transfer entropy maximization

We reanalyze the learning rule in terms of the **transfer entropy (TE)** **[Schreiber, 2000]**

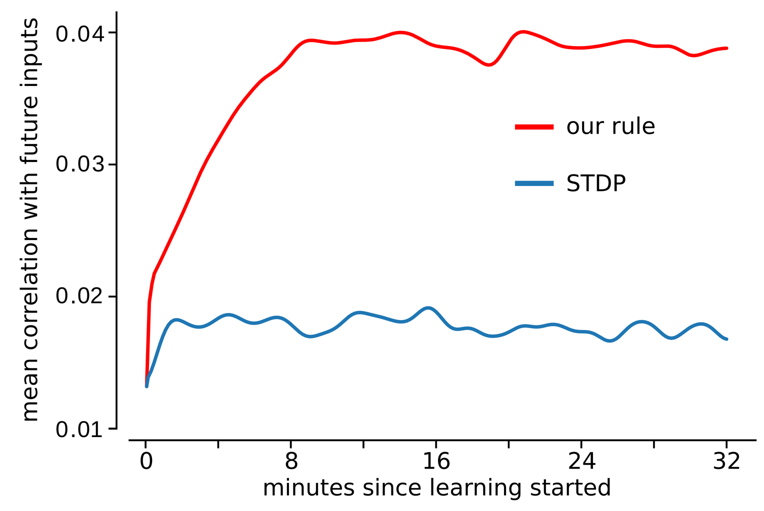
$$T_{\mathbf{z}\to\mathbf{x}} = H\left(\mathbf{x}^t\,\middle|\,\mathbf{x}^{1:t-1}\right) - H\left(\mathbf{x}^t\,\middle|\,\mathbf{x}^{1:t-1},\mathbf{z}^{1:t-1}\right)$$

where $H\left(\mathbf{x}^t\,\middle|\,\mathbf{x}^{1:t-1}\right)$ is here the conditional Shannon entropy.

We derive learning rules that maximize TE for the sequence learning model and find the same form

$$\Delta w_{ij}^t = \alpha_{ij}^t \times \hat{r}_{ij}^t - \beta_{ij}^t$$

but here weight updates are applied under the idealized assumption that post-synaptic spikes are generated to minimize prediction errors.



Conclusion

- We derive learning rules for a recurrent spiking network model from the goal to minimize the free energy of a predictor for future high-dimensional input sequences.
- The emerging learning rules are local, resemble experimentally found plasticity mechanisms and promote the formation of stable neural assembly sequences that become active in synchrony with afferent inputs.
- We analyze the learning rules for prediction error minimization using information theoretic tools and establish a link to maximizing the transfer entropy in the network.
- Our results provides new insights into the mechanisms that enable stable assembly sequence formation in spiking networks.

Acknowledgements

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