Phase synchronization and Information transfer between coupled bursty-oscillatory neural networks in the gamma band

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Introduction
Gamma band oscillations recorded in vivo appear usually as discrete events of synchrony called gamma bursts. The overall process is a random signal close to a filtered noise than a coherent oscillation [1]. Despite its randomness, it has been shown that such bursty gamma oscillations are good candidates for inter-areal brain communication [2]. The communication function implied synchronization between connected bursty gamma oscillatory brain areas. Here we investigated phase synchronization and information transfer between identical connected networks of PING type. More specifically we derived envelope-phase equations of the two identical networks. The phases dynamics derived analytically allow to show the parameters responsible for synchronization. We found that noise and propagation delay between the networks induce Out-of-phase locking bistability in the correlation function between the phase dynamics of the coupled networks, whereas zero propagation delay always leads to In-phase locking. We computed delayed mutual information between phase dynamics of the two connected networks and showed that the type of phase locking (In-phase or Out-of-phase) was in agreement with the structure of mutual information. Our presence of propagation delay was able to allow for two distinct routes of communication similar to what has been shown in other work [2].

Synchronization in gamma bursty networks
Networks of excitatory (M, black) and (D, red) neurons with two-state dynamics (active and quiescent) show oscillations in the gamma band (30-90 Hz) due to stochastically recurring epochs of high synchrony, i.e. Gamma bursts.

From Poisson process analysis, $E_i$ and $I_i$ ($i=1,2$) activities are given by the linear stochastic equations (stochastic Wilson-Cowan equations) [3]:

\[
\begin{align*}
\frac{dx_i}{dt} &= -x_i + E_i(t) + I_i(t) \\
\frac{dI_i}{dt} &= -I_i + \text{sat}(x_i) \\
\end{align*}
\]

The deterministic analogue in the thermodynamic limit admit a fixed point with damped oscillations, therefore the system above admits a quasi cycle attractor.

Envelope and phase equations for $x_i$ and $I_i$

We are interested in the fluctuations from the baseline activities LNA[3]:

\[
\begin{align*}
V_i(t) &= \frac{1}{\sqrt{T}} \int_{t}^{t+T} E_i(t) \, dt \\
\end{align*}
\]

Where $E_i$ and $I_i$ are the deterministic fixed point activities.

Stochastic Averaging Method [3] leads to an envelope-phase representation in terms of key parameters $(\nu, \lambda, \alpha, \alpha', \beta, \phi, \phi', \tau, \tau', \delta, \delta')$

\[
\begin{align*}
\frac{dx_i}{dt} &= -x_i + M(x_i - \theta_i) - \phi(x_i) + \phi'(x_i) \\
\frac{dI_i}{dt} &= -I_i + \text{sat}(x_i) \\
\end{align*}
\]

Then

- Gamma Envelope-phase dynamics for the coupled networks obtained from the LNA and SAM show epochs of phase-locking.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Top: Rasters. Bottom: Activities. Coupled networks show epochs of synchronized activities during gamma bursts. Only oscillatory neurons are shown.}
\end{figure}

Phase synchronization: inter-areal delay induces Out-of-phase locking

- Phase difference dynamics without delay ($\alpha = 0$).

\[
\begin{align*}
\frac{d\phi}{dt} &= -C_{E} + C_{I} \sin(\theta) + \phi(t) - \phi(t - \tau) \\
f(\phi) &= -C_{I} + C_{E} \sin(\theta) \\
\end{align*}
\]

The only deterministic stable solutions are $(\phi = 2\pi k, k = 0,1,2,...)$ In-phase locking only.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Envelopes and phases LFPs time series for the $E_i$ (blue) and $I_i$ (red) populations. (Top) Envelope time-series. (Middle) Phases time-series. (Bottom) LFPs time-series corresponding to the envelopes (top) and phases (middle) time-series}
\end{figure}

- Phase difference dynamics with delay ($\alpha \neq 0$).

\[
\begin{align*}
\frac{d\phi}{dt} &= -C_{E} + C_{I} \sin(\theta) + \phi(t) - \phi(t - \tau) \\
f(\phi) &= -C_{I} + C_{E} \sin(\theta) \\
\end{align*}
\]

The only deterministic stable solutions depending on the values of $\alpha$, $C_{E}$ and $C_{I}$ are $(\phi = 2\pi k, k = 0,1,2,...)$ and Anti-phase locking only.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{The coupled networks with zero propagation delay always exhibit In-phase locking. Left: Correlation function between the phase dynamics. Vertical black bars correspond to $T = 70000$ ms, where $\tau$ is the period of the oscillation. The solid and dashed red lines represent respectively the cases where the phases are obtained via the Hilbert transform performed on real nonlinear dynamics and from the SAM. Right: Stability analysis of the phase difference dynamics. The phase difference dynamics admits a single stable fixed point at zero observed on both stochastic and determinative functions. The deterministic function is obtained analytically after some approximations. For the stochastic function, we computed at each time step a stochastic function $\hat{F}(\phi)$ and then fitted $\hat{F}(\phi)$ with a polynomial function to obtain $f(\phi)$.}
\end{figure}

- Directionality for Information transfer

\[
\begin{align*}
\mathcal{M}(\phi, \psi) &= \sum \mathcal{P}(\theta, \phi - \psi) \sin(\theta) \\
\end{align*}
\]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Phase difference delay and noise cooperate to induce Out-of-phase locking differently to Anti-phase locking. Left: We observe two symmetric peaks in the correlation function, the positions of the peaks are different to $T = 70000$ and $T = 200000$. Right: Phase difference delay and noise cooperate to induce Out-of-phase locking, which is crucial for bidirectional communication.}
\end{figure}

- Delayed Mutual Information for coupled networks with an inter-areal propagation delay

\[
\begin{align*}
\mathcal{M}(\phi, \psi) &= \sum \mathcal{P}(\theta, \phi - \psi) \sin(\theta) \\
\end{align*}
\]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Information transfer through delayed mutual information analysis.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Delayed Mutual Information with inter-areval propagation delay shows a directionality and therefore the possibility for a feedback interregional communication. The phase delayed mutual information curve shows two symmetric peaks positions different from zero. This two peaks correspond respectively to the situations where one of the networks is the leader while the other is the lagged and vice versa. This suggests two routes for information transfer. The exact value of the delay would be a set to $\tau = 0$.}
\end{figure}

Conclusion
We have derived the envelope-phase dynamics of a coupled bursty oscillatory networks in the gamma band. Our decomposition allows to relate phase synchronization on gamma bursts. In last, phase synchronization is a transient process which happens more efficiently during bursts of the processes. In addition we have shown that inter-areal propagation delay and noise cooperate to induce out of phase locking which is crucial for bidirectional communication. Our study suggests that bursty oscillations which are ubiquitously observed in vivo are good candidates for flexible brain communication. Future studies will have the ambition to extend our study to more than two networks with heterogeneities.

Acknowledgments
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References