Topological analysis of multi-site LFP data

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Neural Events

LFP originates as a weighted sum of synaptic and somato-dendritic currents. The weight monotonically decreases with distance to the electrode. Intervals of synchronized firing are seen as waveforms - *neural events* - in the recorded time series. Classically, fine-grained properties of typical waveforms and their temporal distribution have been used to identify global brain states (see e.g. for PGO-waves in stages of sleep). An elaborate analysis of LFP neural events can give a more precise description of a global brain state. Some of them have been studied in great detail (in addition to e.g. Locus Ceruleus in the Pons, as well as to PGO-waves in Lateral Geniculate Nucleus of the Thalamus, see sleep spindles in the Thalamus and areas of the Cortex, Sharp-Wave-Ripples in the Hippocampus and K-complexes). A common set of statistical properties of neural events could be used for systems-level description of dynamic global brain signalling.

Towards Brain State Characterization

We look at brain state through mesoscale spontaneous activity (LFP) at multiple sites simultaneously, focusing on neural events – temporary increases in LFP power that are localized in frequency. One initial goal is to obtain descriptive features of events that are robust to recording noise and partially invariant to its length.

1 Introduction to Persistent Homology

Suppose the sample event is represented as a point cloud $L \subset \mathbb{R}^n$, and consider a distance function $\psi : \mathbb{R}^n \to \mathbb{R}$ with $\psi(x) = d(x, L)$, evaluating the minimal distance from points in L to a point $x \in \mathbb{R}^n$, e.g. as $min_{l \in L}||l - x||_2$. For each nonnegative fixed parameter $r \geq 0$, we can look at homology of a space formed by the union of balls of radius r, or, equivalently, homology of a sublevel set $Y_r = \psi^{-1}[0, r]$ of the function ψ. As we increase parameter r, we have inclusion of sublevel sets, i.e. for each $r' \leq r''$ we get $Y_{r'} \subseteq Y_{r''}$; yet only a finite number of sublevel sets differ in homology. *Persistent Homology* canonically tracks the change in homology of sublevel sets (in this setting) as parameter r increases. For example (visualization borrowed from a review by Edelsbrunner and Morozov), left shows an example space and a height function, where the marked critical points denote the change in homology. More generally, it is defined on a *filtration* - a sequence of nested spaces, where an inclusion induces a map on homologies of spaces, indexed by a set of parameter values r^i. In the resulting sequence of linear maps homology classes of space Y_{r^j} are mapped to homology classes of space Y_{r^k}:

$$g_\star : 0 \to H_n(Y_{r^1}) \to H_n(Y_{r^2}) \to ... \to H_n(Y), \tag{1}$$

which means that some of them may be mapped to 0. Hence the image of $H_n(Y_{r^j})$ in $H_n(Y_{r^k})$ forms the *persistent homology group* whose rank shows the number of homology classes "present" at Y_{r^j}, but not at Y_{r^k}. In the sublevel set example - two connected components get merged into a single component as parameter r passes a critical point and one homology class disappears. Recorded values of appearing and disappearing homology classes with a filtration parameter increase are called *birth* an *death* times, where tracking changes in H_0 corresponds to tracking "clusters", in H_1 – to tracking "loops" or "1-dimensional" holes etc. These changes are visualized by a *persistence diagram*. In practice a filtration can be obtained from sublevel sets of scalar-valued function, or from a point cloud.

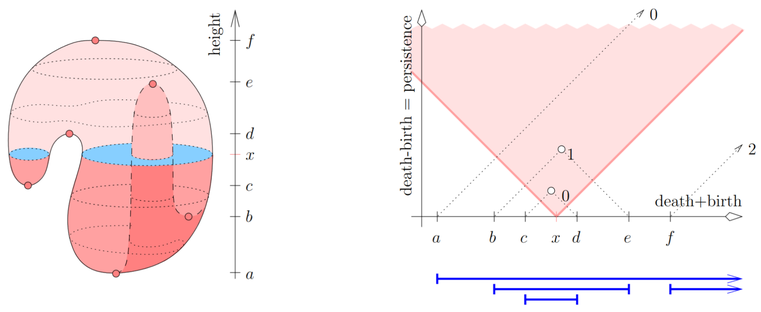

Figure 1: Example figure from Edelsbrunner, Morozov (2013) *Persistent Homology: Theory and Practice.*

2 Filtration of a Point Cloud of Delay Embedding

A point cloud is constructed using delay embedding so that persistence of it's filtration can be computed. Changes of waveform amplitude within duration of an event can be thought of as a discriminative feature, relative to constant amplitude waveform. Because both represent an increase in power in one or more frequencies, the barcodes can be used to derive features representing periodicity. Events of different duration can still have similar underlying dynamics, i.e. two filtrations of two reconstructed trajectories may be treated as similar, even if measured for different time intervals.

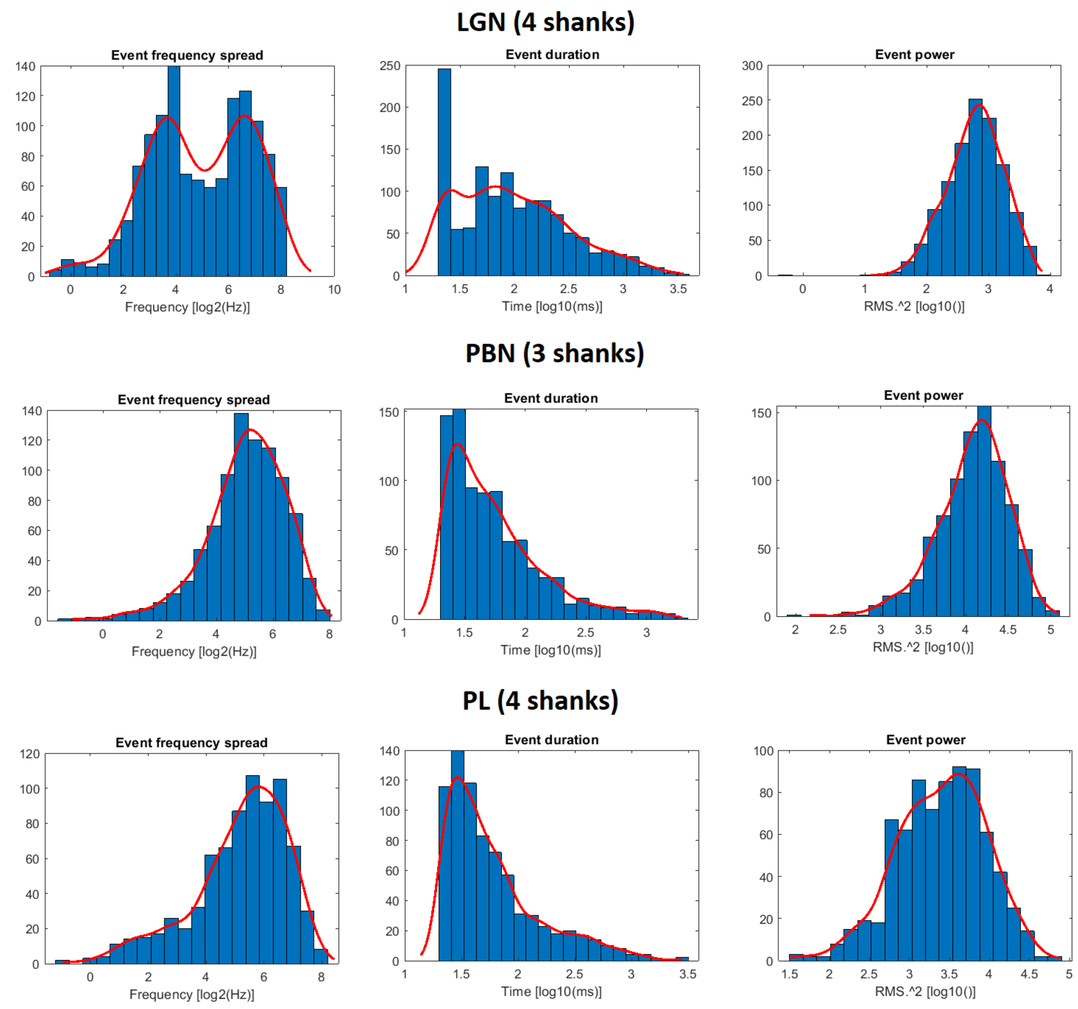

Figure 2: Coarse look at neural events: histograms of frequency range (spread), duration and event power for Lateral Geniculate Nucleus (LGN) of the Thalamus, Peribrachial Nucleus (PBN) of the Pons and Pyramidal Layer (PL) of the Hippocampus.

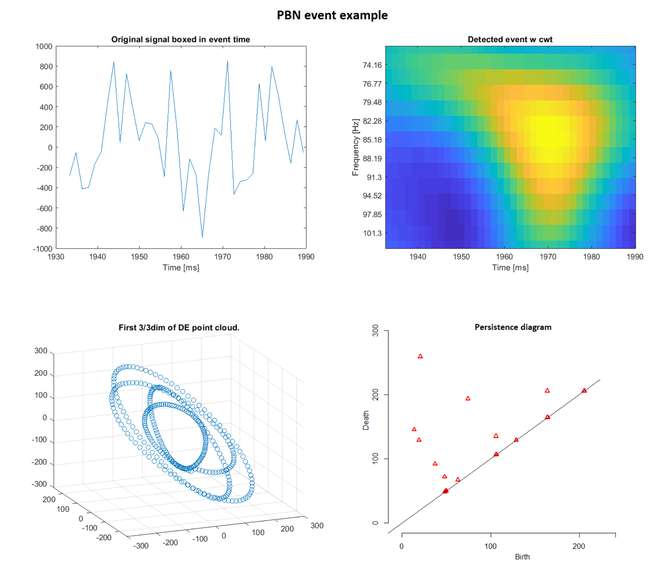

Figure 3: Example event from PBN (clockwise from top): localization in time, wavelet representation localized in frequency, persistence diagram of the filtration of a point cloud, point cloud obtained via delay embedding. On the persistence diagram, red triangles correspond to cycles or underlying periodicities in an oscillation. Absence of cycles further from diagonal indicates absence of a single prominent oscillation.

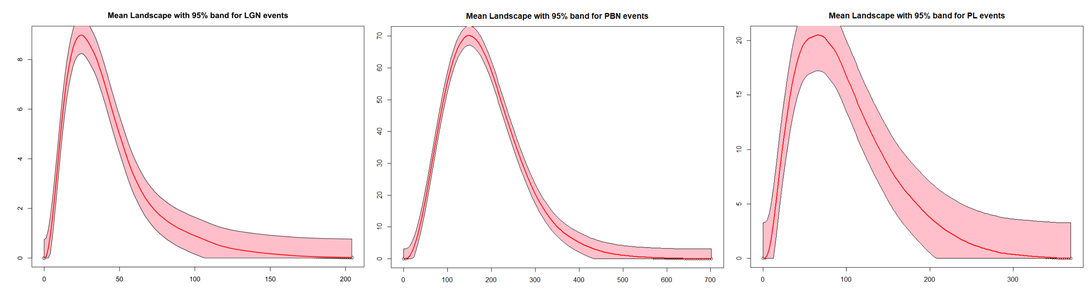

Figure 4: Mean persistence landscapes of events at PBN, LGN and PL brain sites with confidence band. Persistence landscape realizes a maps from a space of persistence diagrams to a function space, permitting statistical analysis.

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