Action potential propagation in long-range axonal fibre bundles

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Introduction

Background

- Recent advances in MRI imaging make it possible to determine white matter structure non-invasively
- Action potential velocity determines delays between brain regions
- Standard numerical techniques to model the dynamics of action potentials require huge computational effort

Aims

- Develop a computationally efficient framework to compute action potential velocity (spike-diffuse-spike model)
- Determine how action potential velocity depends on structural parameters
- Study interaction of action potentials between ephaptically coupled fibres

Methods

- Axonal membrane potential V is described by cable equation

$$\tau\frac{\partial V}{\partial t} = \lambda^2\frac{\partial^2 V}{\partial t^2} - V + R_m I_{chan}(V,t)$$

- We model channel currents I_{chan} as threshold-activated events (Fig. 1)
- The resulting linear cable equation can be solved explicitly

$$V(x,t) = \sum_{n=1}^{N}\Phi_I(x+nL, t+nt_{sp}) \qquad \Phi_I(x,t) = R_m\int_0^t I_{chan}(t-s)G(x,s)\mathrm{d}s$$

- Unknown time delay t_{sp} is computed numerically from

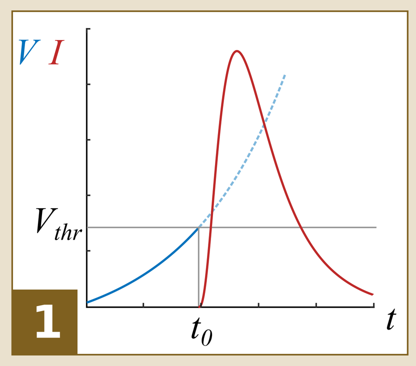
$$V_{thr} = \sum_{n=1}^{N}\Phi_I(nL, nt_{sp})$$

- Velocity is given by $v = L / t_{sp}$ (Fig. 2)
- Results are compared with detailed numerical model [1]
- Methods can be extended to axonal fibre bundles (Fig 3):

$$\tau\frac{\partial(V_n - V_e)}{\partial t} = \lambda_n^2\frac{\partial^2 V_n}{\partial x^2} - (V_n - V_e) + R_m I_n^{chan}(t)$$

- Using assumptions from [2] and $U_n = V_n - V_e$

$$\tau\frac{\partial U_n}{\partial t} = \lambda^2\frac{\partial^2 U_n}{\partial x^2} - U_n + R_m I_n^{chan}(t) - \alpha\lambda_n^2\sum_p R_{ax,p}^{-1}\frac{\partial^2 U_p}{\partial x^2}$$



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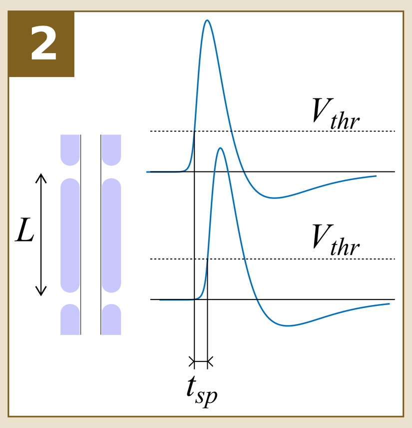

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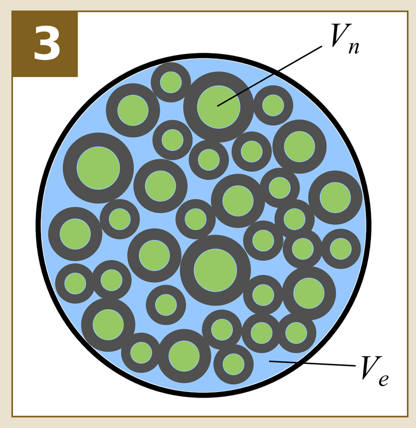

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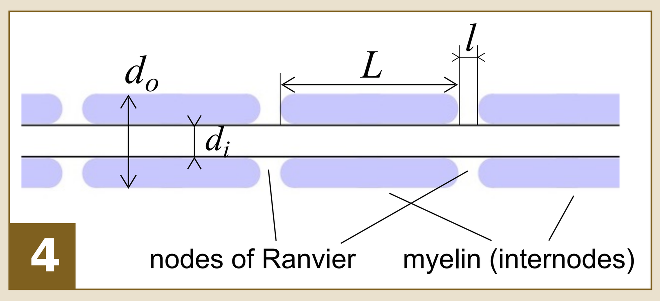

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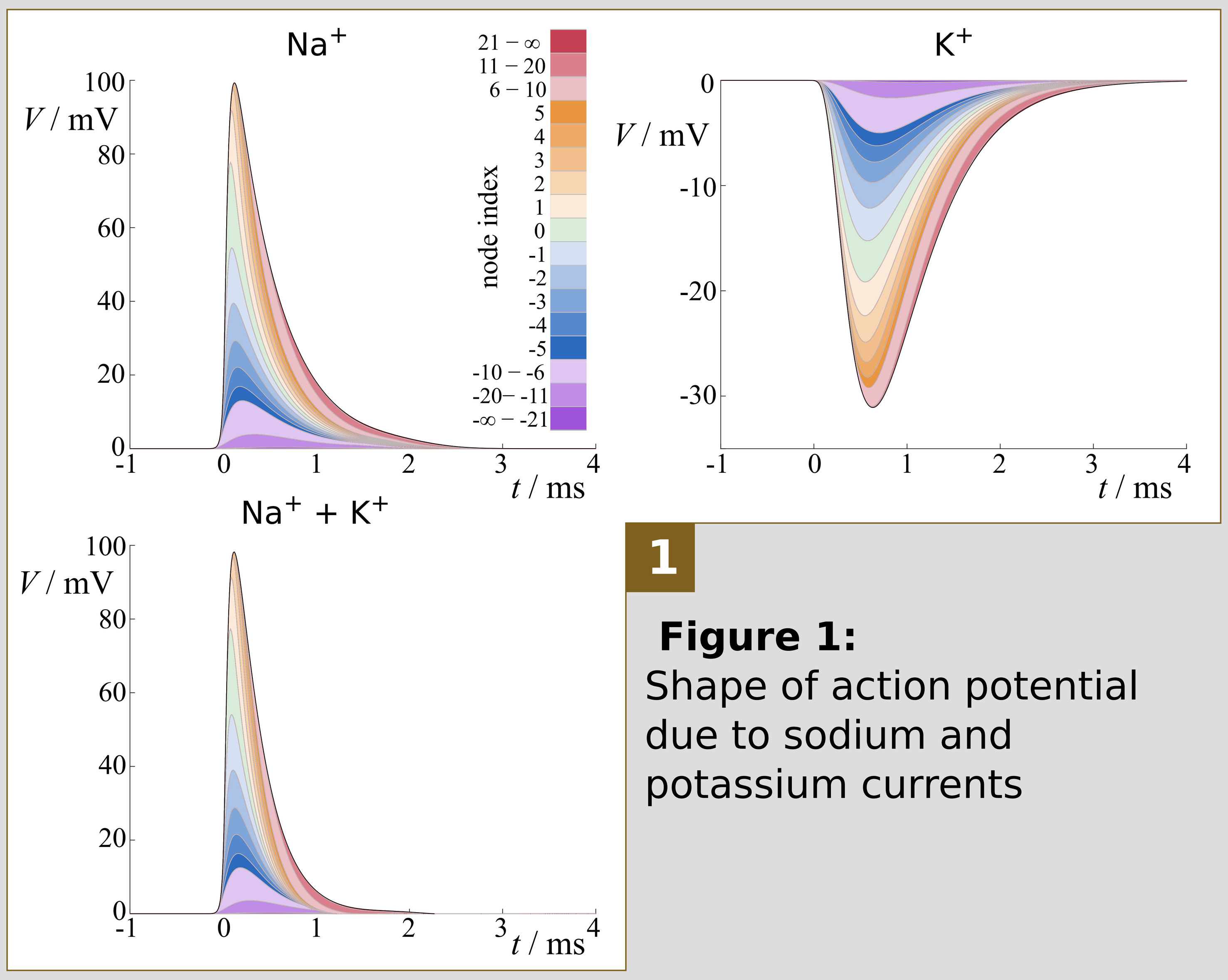
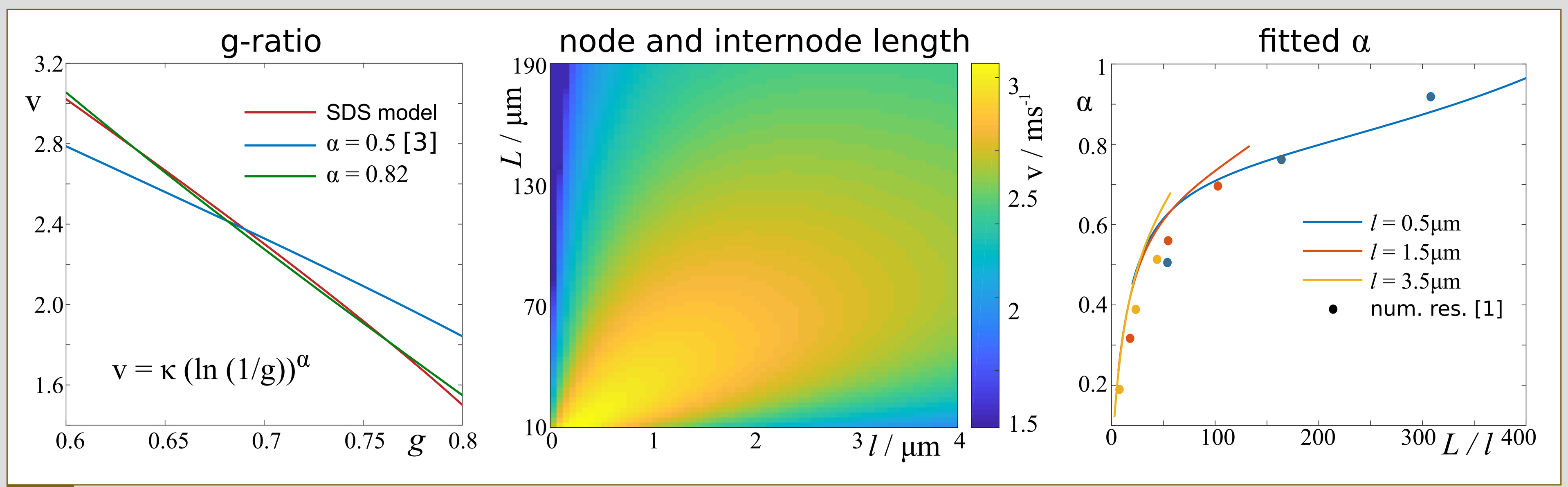
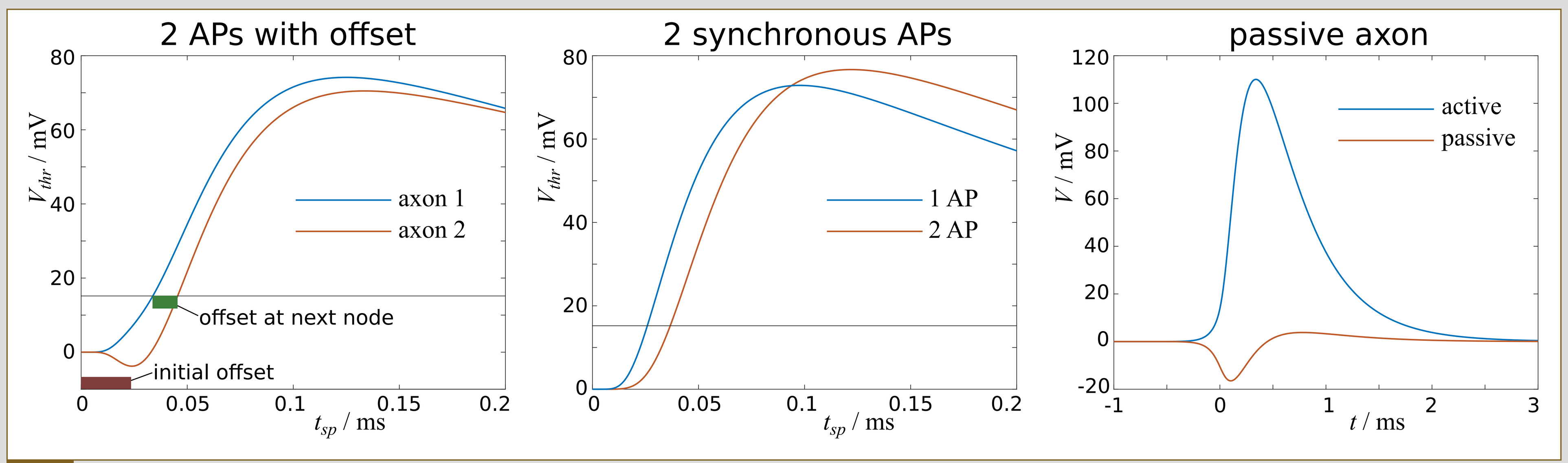
Results



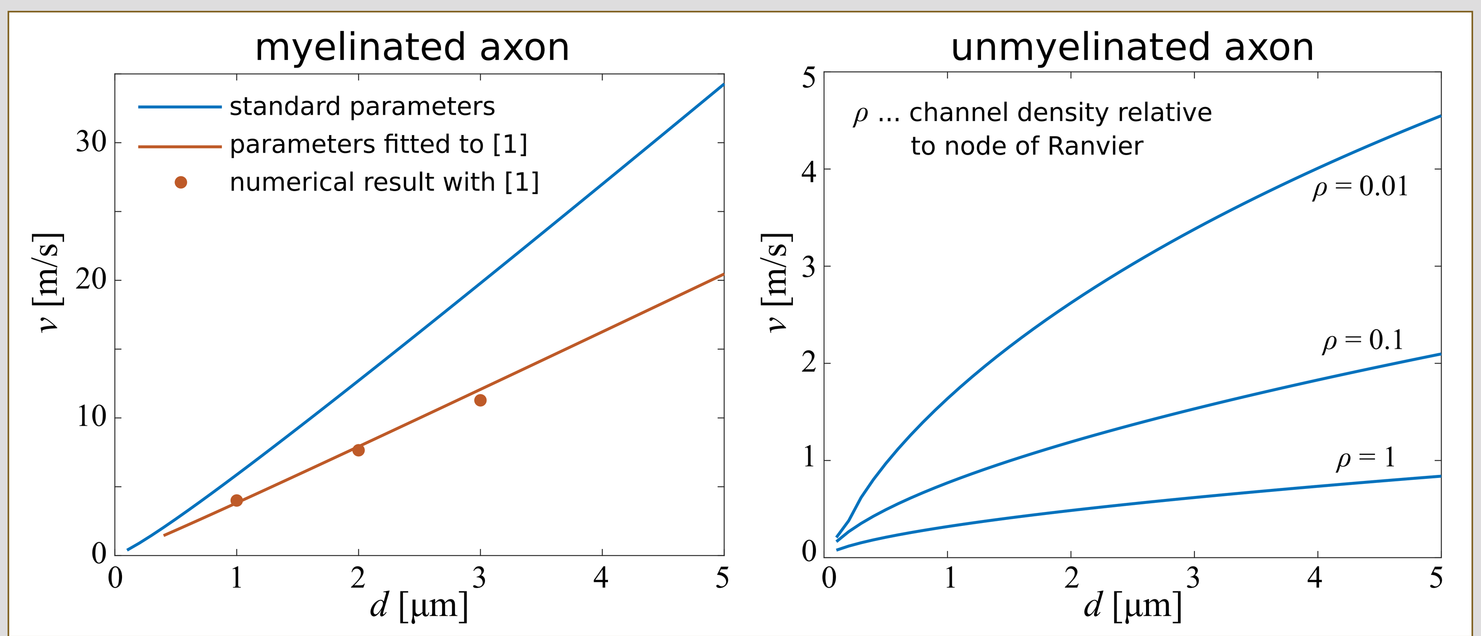
1 **Figure 1:** Shape of action potential due to sodium and potassium currents



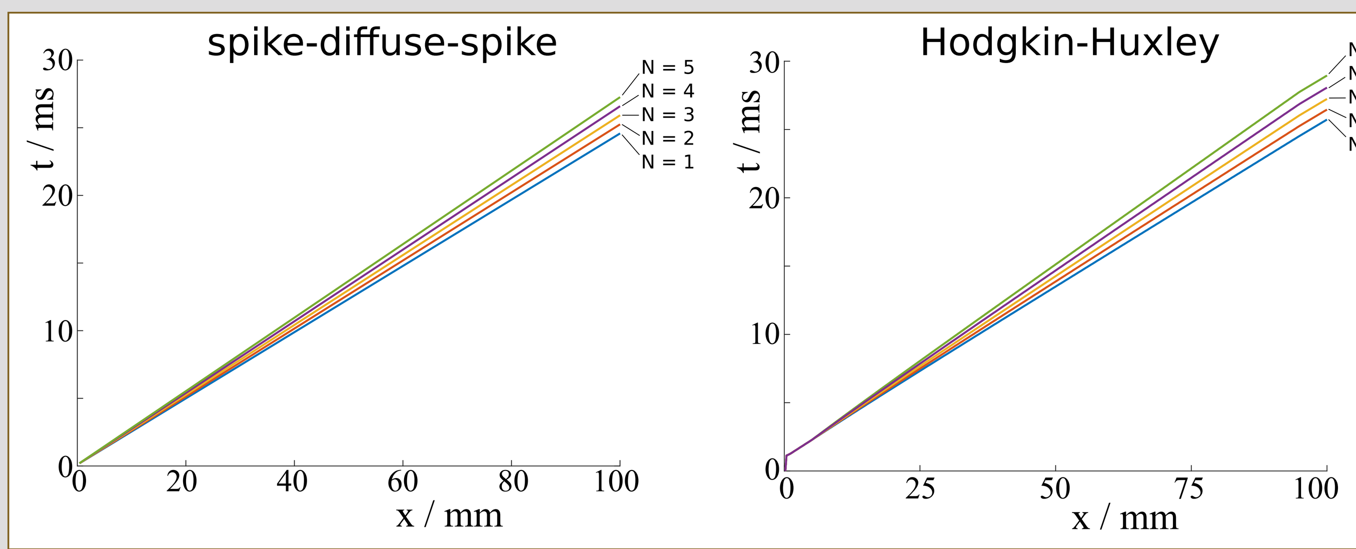
3 **Figure 3:** Propagation velocity and myelin structure



4 **Figure 4:** Action potential dynamics in a coupled pair of identical axons



2 **Figure 2:** Propagation velocity and axon diameter



5 **Figure 5:** Trajectories of spike volleys (N spikes) in fibre bundle with finite extra-axonal space

Conclusion

single axon

- We use a implified description of action potential propagation without explicit Hodgkin-Huxley dynamics
- Approach yields an analytical expression for the (implicit) relationship between action potential velcoity and model parameters
- Many known results are reproduced qualitatively and quantitatively
- velocity depends more strongly on g-ratio than inferred in [3]
- Speed-up by three orders of magnitude in comparison to numerical integration of cable equation

axon bundle

- Ephaptic coupling due to limited extra-axonal space synchronises and slows down action potentials (as observed by [2] and [4])
- Ephaptic coupling leads to activity-dependent delays

Acknowledgements

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References

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