Action potential propagation in long-range axonal fibre bundles

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Introduction

Background

• Recent advances in MRI imaging make it possible to determine white matter structure non-invasively
• Action potential velocity determines delays between brain regions
• Standard numerical techniques to model the dynamics of action potentials require huge computational effort

Aims

• Develop a computationally efficient framework to compute action potential velocity (spike-diffuse-spike model)
• Determine how action potential velocity depends on structural parameters
• Study interaction of action potentials between ephaptically coupled fibres

Methods

• Axonal membrane potential $V$ is described by cable equation
  \[ \frac{\partial V}{\partial t} = \lambda I^{\text{mem}}(V, t) + \frac{\partial^2 V}{\partial x^2} \]
• We model channel currents $I^{\text{mem}}$ as threshold-activated events (Fig. 1)
• The resulting linear cable equation can be solved explicitly
  \[ V(x, t) = \sum_{n=1}^{N} \Phi_n(I_n(t) + nL, t + n\tau_p) \]
• Velocity is given by $v = L / t_p$ (Fig. 2)
• Results are compared with detailed numerical model [1]
• Methods can be extended to axonal fibre bundles (Fig. 3):
  \[ \frac{\partial (V_n - V_0)}{\partial t} = \lambda \frac{\partial V_n}{\partial x^2} - (V_n - V_0) + \rho \alpha \frac{\partial U_n}{\partial x} \]
• Using assumptions from [2] and $U_n = V_n - V_r$

Results

Figure 1: Shape of action potential due to sodium and potassium currents

Figure 2: Propagation velocity and axon diameter

Figure 3: Propagation velocity and myelin structure

Figure 4: Action potential dynamics in a coupled pair of identical axons

Figure 5: Trajectories of spike volleys (N spikes) in fibre bundle with finite extra-axonal space

Conclusion

single axon

• We use a simplified description of action potential propagation without explicit Hodgkin-Huxley dynamics
• Approach yields an analytical expression for the (implicit) relationship between action potential velocity and model parameters
• Many known results are reproduced qualitatively and quantitatively
• Speed-up by three orders of magnitude in comparison to numerical integration of cable equation

axon bundle

• Ephaptic coupling due to limited extra-axonal space synchronizes and slows down action potentials (as observed by [2] and [4])
• Ephaptic coupling leads to activity-dependent delays

References


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